DO NOW

Review the information from Section 1.5 in your notes on Inverse Functions.

3.6 Derivatives of Inverse Functions

Recall information about inverse functions:

* Switch x and y to rewrite
* reflect over $y=x$
* range $\leftrightarrow$ domain (switch)
* $f(g(x))=x$ and $g(f(x))=x$
* horizontal line test for inverse to exist
* May or may not have an inverse
* monotonic (strictly increasing or decreasing)
* If (a,b) is in the function, then (b,a) is in the inverse
* Inverse trig function graphs - pg 42
* Trig value tables

Continuity and Differentiability of Inverse Functions

Let *f* be a function whose domain is an interval *I*. If *f* has an inverse function, then the following statements are true.

1. If f is continuous on its domain, f^{-1} is continuous on its domain.
2. If f is increasing on its domain, f^{-1} is increasing on its domain.
3. If f is decreasing on its domain, f^{-1} is decreasing on its domain.
4. If f is differentiable at c and $f'(c)\neq 0$ then f^{-1} is differentiable at c.

The Derivative of an Inverse Function

Let *f* be a function that is differentiable on an interval *I*. If *f* has an inverse function, then *g* is differentiable at any *x* for which $f'(g(x)) \neq 0$. Moreover:

$$g'(x)=\frac{1}{f'(g(x))}, \quad f'(g(x))\neq 0$$

OR

$$(f^{-1})'(x)=\frac{1}{f'(f^{-1}(x))}$$

* Is the reciprocal of the derivative of the original function.

Example: pg 179; #9

Show that the slopes of the graphs of *f* and f^{-1} are reciprocals at the indicated points.

$f(x)=\sqrt{x-4}$ $(5, 1)$

$f^{-1}(x)=x^2+4,\ x\geq 0$ $(1, 5)$

$f(x)=\sqrt{x-4}$

$f'(x)=\frac{1}{2}(x-4)^{-1/2}$

$f'(x)=\frac{1}{2\sqrt{x-4}}$

$f'(5)=\frac{1}{2\sqrt{5-4}}$

$f'(5)=\frac{1}{2}$

$f^{-1}(x)=x^2+4$

$(f^{-1})'(x)=2x$

$(f^{-1})'(1)=2(1)$

$=2$

$\frac{1}{2}$ is the reciprocal of $\frac{2}{1}$.

Derivatives of Inverse Trigonometric Functions

Let *u* be a differentiable function of *x*.

$$\frac{d}{dx}[\arcsin u]=\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arccos u]=-\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u]=\frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arccot}\ u]=-\frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec}\ u]=\frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}[\text{arccsc}\ u]=-\frac{u'}{|u|\sqrt{u^2-1}}$$

Examples: Find the derivative of the function.

1. $f(x) = \text{arcsec } 3x$ $\quad u = 3x$

$$f'(x) = \frac{3}{|3x|\sqrt{(3x)^2-1}}$$

$$f'(x) = \frac{\cancel{3}}{\cancel{3}|x|\sqrt{9x^2-1}}$$

$$f'(x) = \frac{1}{|x|\sqrt{9x^2-1}}$$

2. $g(t) = \arccos\sqrt{2t-1}$ $\quad u=(2t-1)^{1/2}$

$$g'(t) = \frac{-1}{\sqrt{1-(\sqrt{2t-1})^2}} \cdot \frac{1}{2}(2t-1)^{-1/2}(2)$$

$$g'(t) = \frac{-1}{\sqrt{1-2t+1}} \cdot \frac{1}{\sqrt{2t-1}}$$

$$g'(t) = \frac{-1}{\sqrt{2-2t}\,\sqrt{2t-1}}$$

3. $f(x) = (\arcsin x)^2$ $\quad y = u^2$ $\quad u = \arcsin x$

$$f'(x) = 2(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{2\arcsin x}{\sqrt{1-x^2}}$$

4. $y = \sec^{-1}\sqrt{1+x^2}$ $\quad u=(1+x^2)^{1/2}$

$$y = \text{arcsec}(1+x^2)^{1/2}$$

$$y' = \frac{1}{|\sqrt{1+x^2}|\sqrt{(\sqrt{1+x^2})^2-1}} \cdot \frac{1}{2}(1+x^2)^{-1/2}(2x)$$

5. $y = \arctan\left(\frac{x}{a}\right) + \ln\sqrt{\frac{x-a}{x+a}}$

HOMEWORK

pg 179 - 180; 7, 8, 10, 11 - 33 odd,
45 - 51 odd, 57